



Metric Geometry and Gerrymandering

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Many slides from M. Duchin & M. Bernstein

MGGG

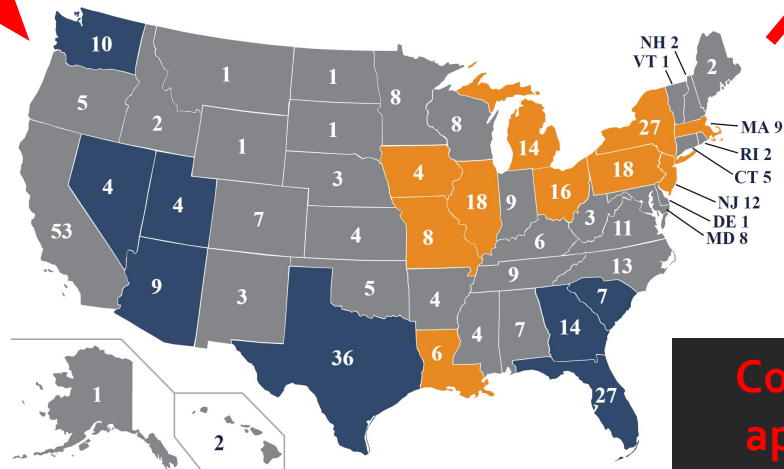
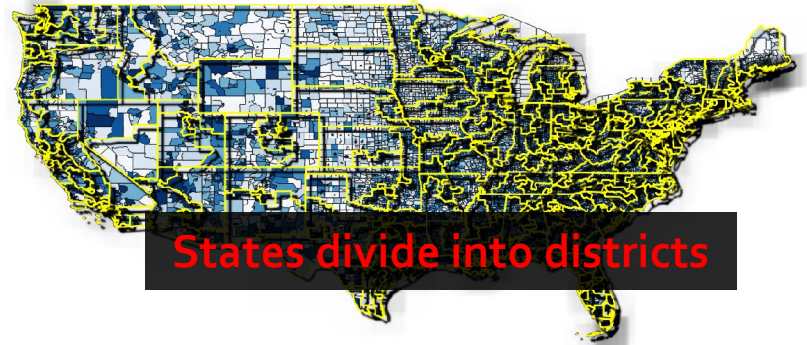
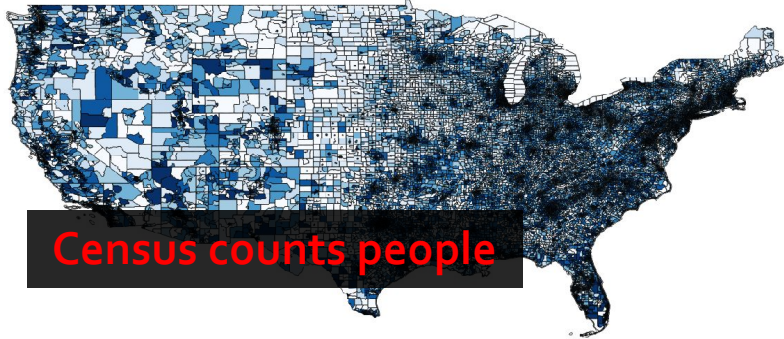
Metric Geometry & Gerrymandering Group

Small team studying applications of math and computing to redistricting

- Research & publication
- Interdisciplinary collaboration
- Outreach & education

sites.tufts.edu/gerrymandr/

Congressional Representation



Congressional reps are apportioned to states

Redistricting as a Math Problem



o has 45% of the population but 25% of the districts
Red has 52.5% of the population by 75% of the districts

... tens of thousands of census blocks per district!

Partitioning with attributes

Goals and Constraints

■ Goals

- **Proportionality:** Districts are representative
- **Gerrymandering:** Partition to extremize an attribute

■ Constraints

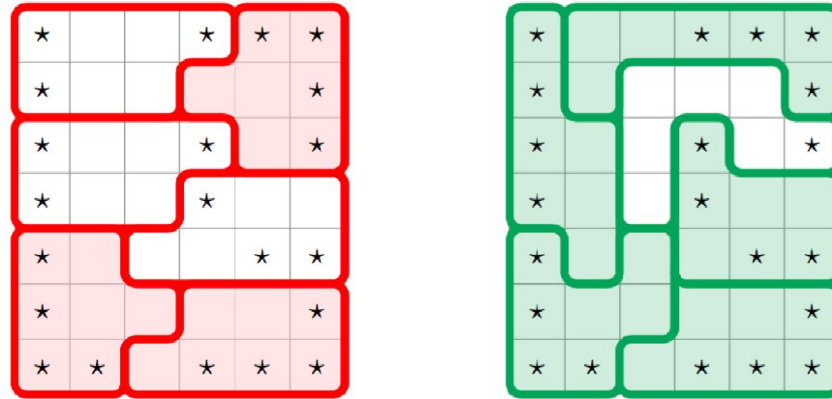
- Equal population
- No holes
- Not too “eccentrically shaped”

Other Values or Principles

- Proportionality
- Competitiveness
- Governability
- Partisan Fairness
- ...?

Relatively few legal parameters

How to Gerrymander



Intuition:

Any agenda will cause eccentric shapes.

Image from:

A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap
Bernstein & Duchin, *Notices of the AMS* (to appear)

Packing & cracking

Compactness

COMPACTNESS

► Many metrics exist:

► Isoperimetry

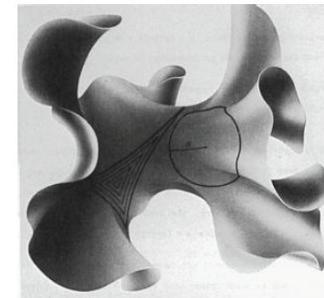
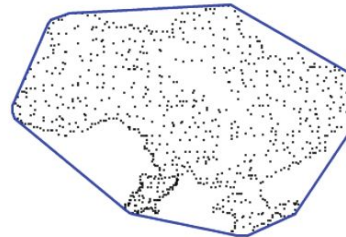
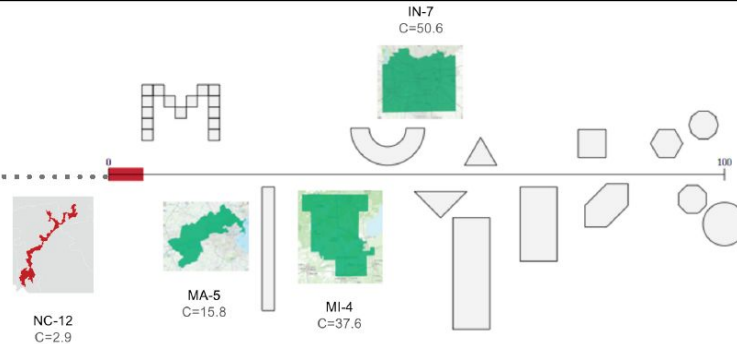
$$0 \leq 400\pi A/P^2 \leq 100$$

► Convexity

How indented?

► Dispersion

How spread out?



but perimeter is problematic

there are many legitimate reasons for non-convexity

and all of this is 19th century mathematics!

NC-12 at the Census Tract Level

Until 2013



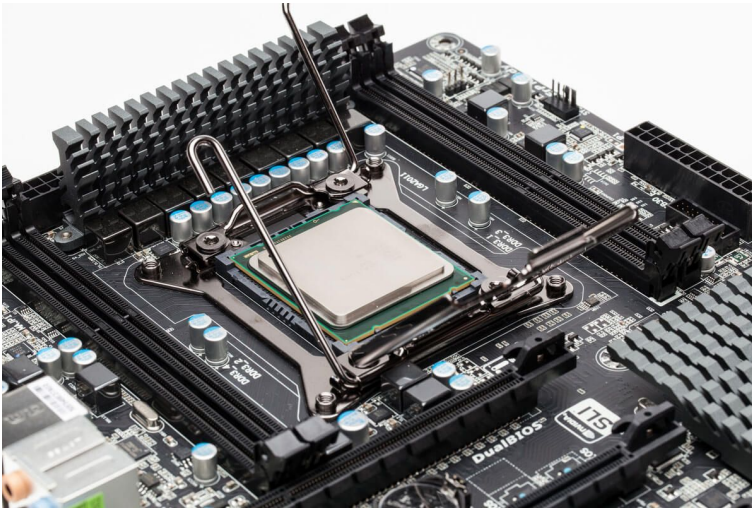
"If you drove down the interstate with both car doors open, you'd **kill most of the people in the district.**"

- Unnamed state legislator
Reported in: "Thomas right to oppose racial 'homelands'"
(*The Item*, August 17, 1994)



What can we do as
programmers and
computer scientists?

Partners in Redistricting



10^9 computations/second
No legal understanding
No sympathy



?? computations/second
Strong legal understanding
Potentially sympathetic

Spectrum

Clearly easy:

- Visualizing districting plans
- Data collection

Clearly difficult:

- Extracting optimal plans

Huge gray area:

- Improving plans
- Evaluating compactness
- Sampling possible plans



Let's be clear:

Any software extracting the “best possible”
districting plan* also resolves the most
famous open problem** in computer science.

$P \stackrel{?}{=} NP$

* under any reasonable metric.

** (News!) Perhaps not open any more.

What Can We Do?

The screenshot displays the DistrictBuilder web application interface. At the top, there are navigation tabs for PLAN, DRAW, SHARE, and LEADERBOARDS. Below these are utility links for About, Resources, Contest Rules, Support, My Account, and Logout. The main interface is divided into a map editing area on the left and a statistics panel on the right. The map editing area includes a toolbar with tools like Pan, Single Select, Rectangle Select, Polygon Select, Anchor District, Assign to District, Click and Drag, and Fix Unassigned. The statistics panel, titled 'Balanced Plan', shows a table of district data and a legend for Hispanic VAP and District status.

Basic Information

Balanced Plan

Score (Target) 10 (of 10)
Target Pop. (152,601) 10 (of 10)
Contiguous 10 (of 10)
Black VAP Majority (>50%) 3 (of 10)
Hisp. VAP Majority (>50%) 0 (of 10)

Dist.	Tot Pop	Contiguous	Compactness
0		✓	n/a
1	153,321	✓	66.21%
2	152,433	✓	48.01%
3	152,127	✓	81.04%
4	153,567	✓	54.34%
5	153,615	✓	61.23%
6	151,742	✓	65.44%
7	152,917	✓	62.68%
8	153,045	✓	73.60%
9	151,838	✓	70.85%
10	151,401	✓	65.49%

Hispanic VAP

- > 447
- > 271
- > 139
- > 55
- < 55

District

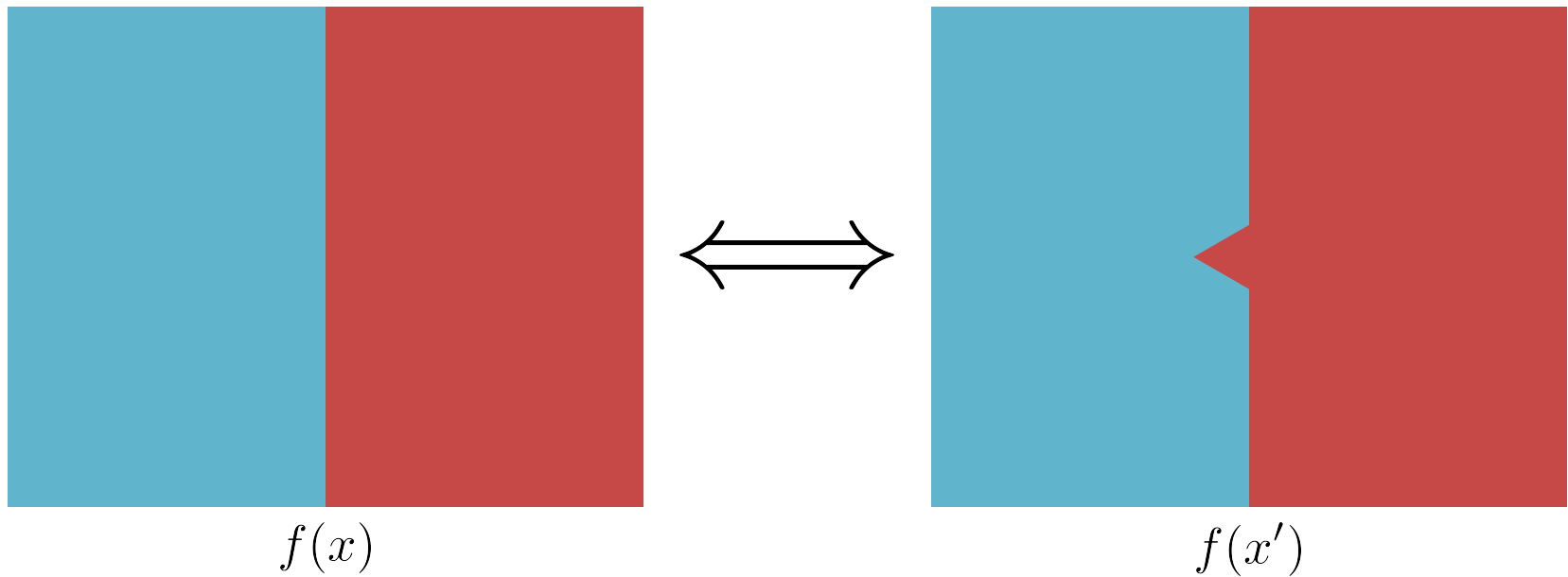
- Far Over Target
- Over Target
- Within Target
- Under Target
- Far Under Target
- Locked For Editing
- Highlighted

Plan Name: Balanced Plan | Currently Viewing: Division | Last Saved on 8/8 at 4:39

DistrictBuilder

Analysis and comparison

What Can We Do?



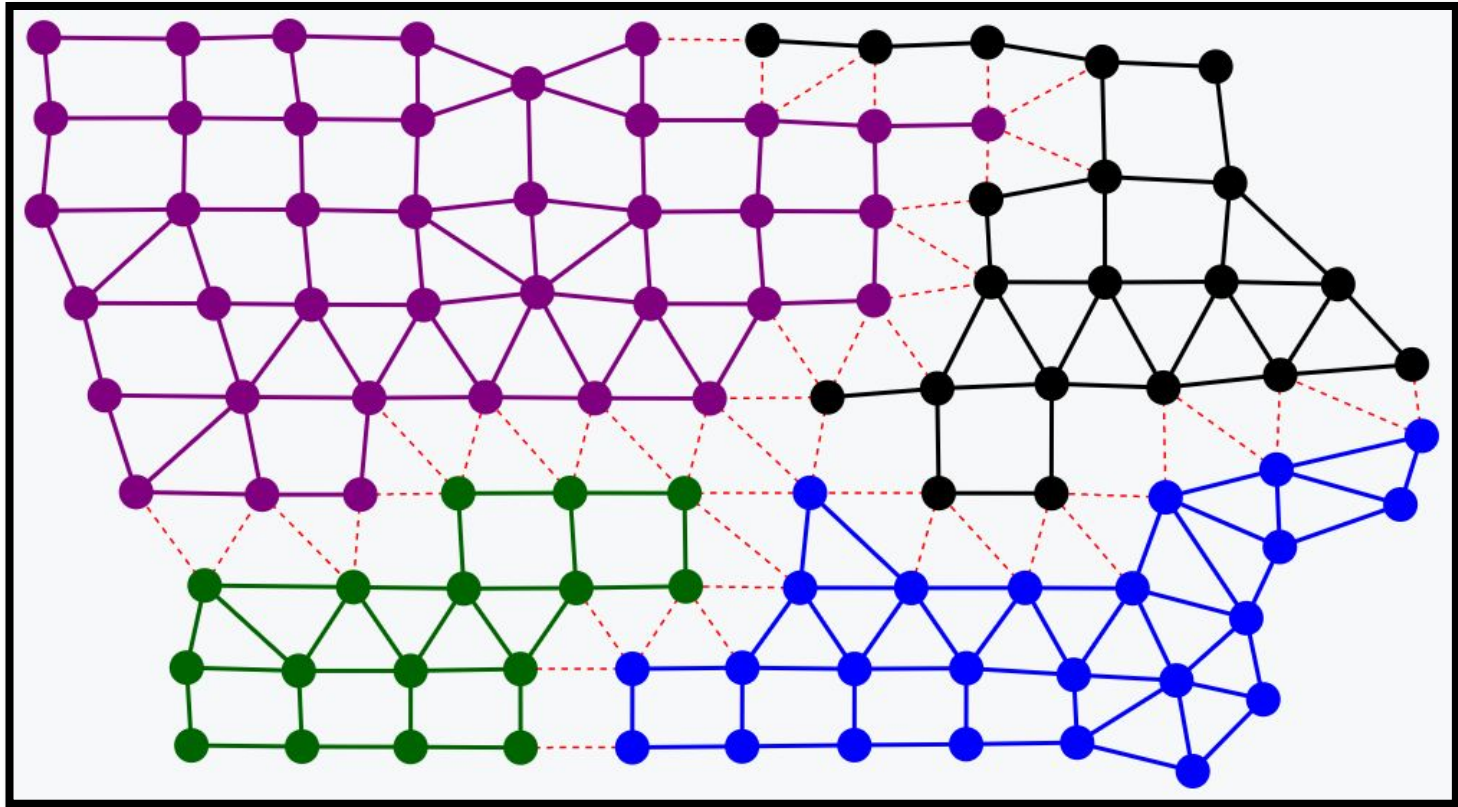
Local optimization

Which Objective Function?

- Isoperimetric ratio?
- Graph curvature?
 - Dispersion?
- Equal population?
- Minority representation?
 - Efficiency gap?

**"Pareto
optimality"**

What Can We Do?



Screenshot from "Quantifying Gerrymandering" (Duke Data⁺)

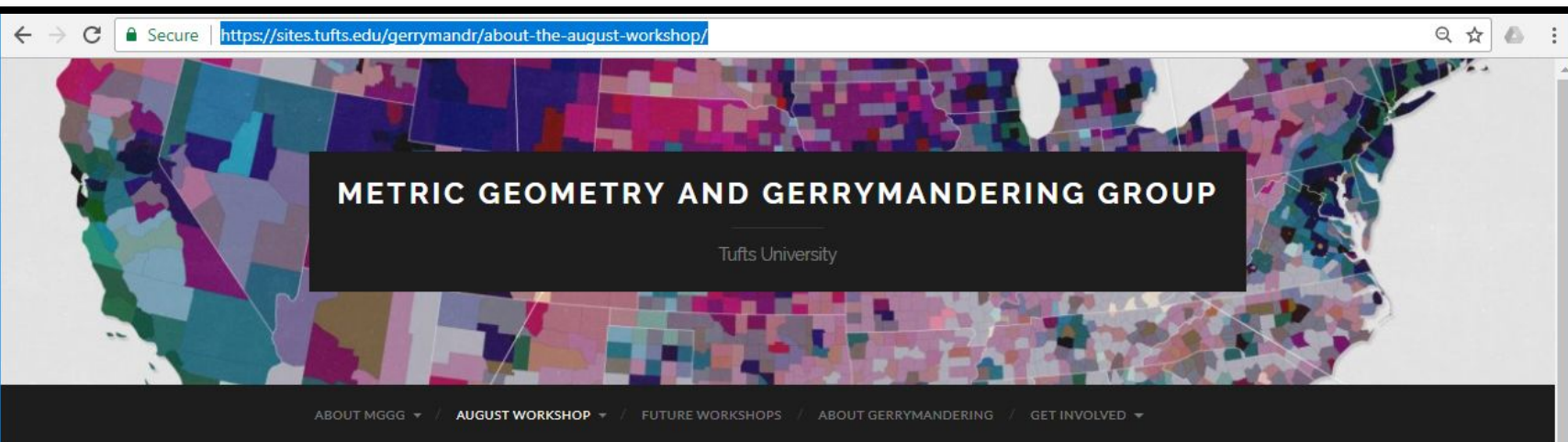
<https://services.math.duke.edu/projects/gerrymandering/>

Sampling/MCMC

Call to action:

We need your help.

MGGG Effort



About the August Workshop

A Geometry of Redistricting workshop will be offered at Tufts University from August 7-11, 2017, mixing math, law, and civil rights. The first three days of the week (M-W) will be open to the public and made [available online](#). The last two days (Th-F) will be devoted to [specialized training](#), broken down into three tracks for which participants were selected by an application process in early Spring.

Registration for the Monday to Wednesday workshop has now closed. We will accommodate walk-ins after pre-registered participants have been seated.

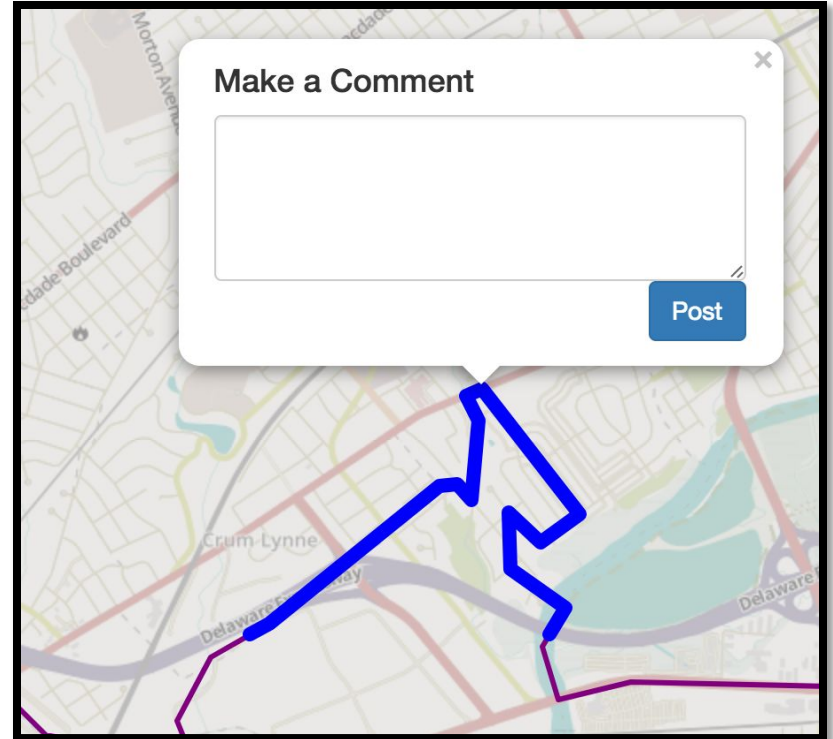
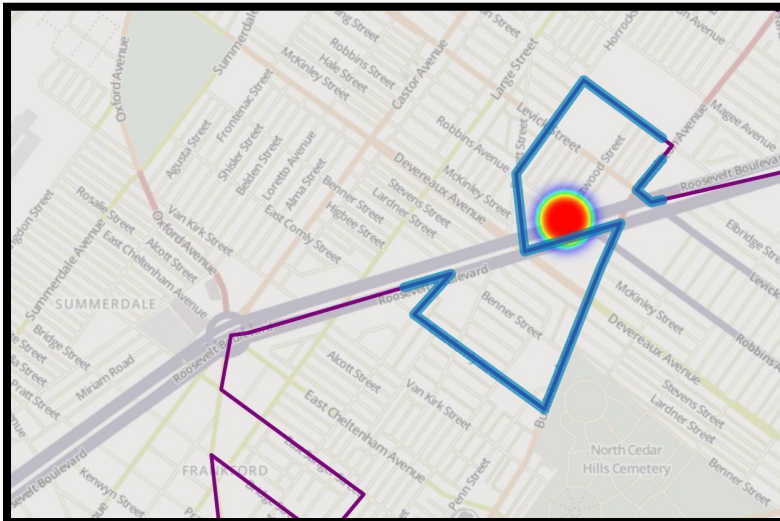
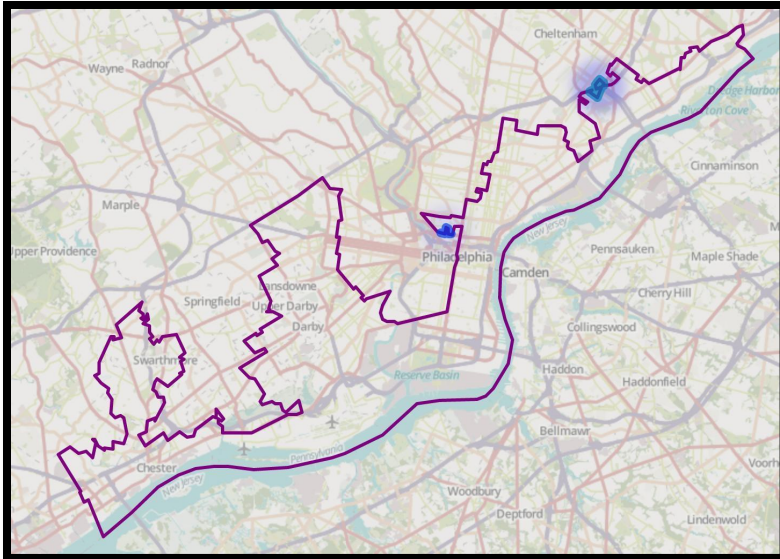
Schedule for Monday-Wednesday

Here is the full [program](#) for Monday-Wednesday, and here is the [schedule](#) on its own.

"GIS Track"

N. Doiron:

DistrictGenius



<https://github.com/gerrymandr/district-genius>

A. M'ndange-Pfupfu
& V. Archambault:

QGIS Compactness Plugin

Compactness Calculator

Select Compactness Metrics:

- Polsby-Popper
- Convex Hull
- Reock
- Schwartzberg

Choose file output path:

Add to map canvas

Python Console

```
101 False
102 False
103 True
104 /home/ariel/temp.geojson
105 Saved to /home/ariel/test.json
106 Saved to /home/ariel/test.shp
107 Saved to /home/ariel/demo.geojson
108
>>>
```

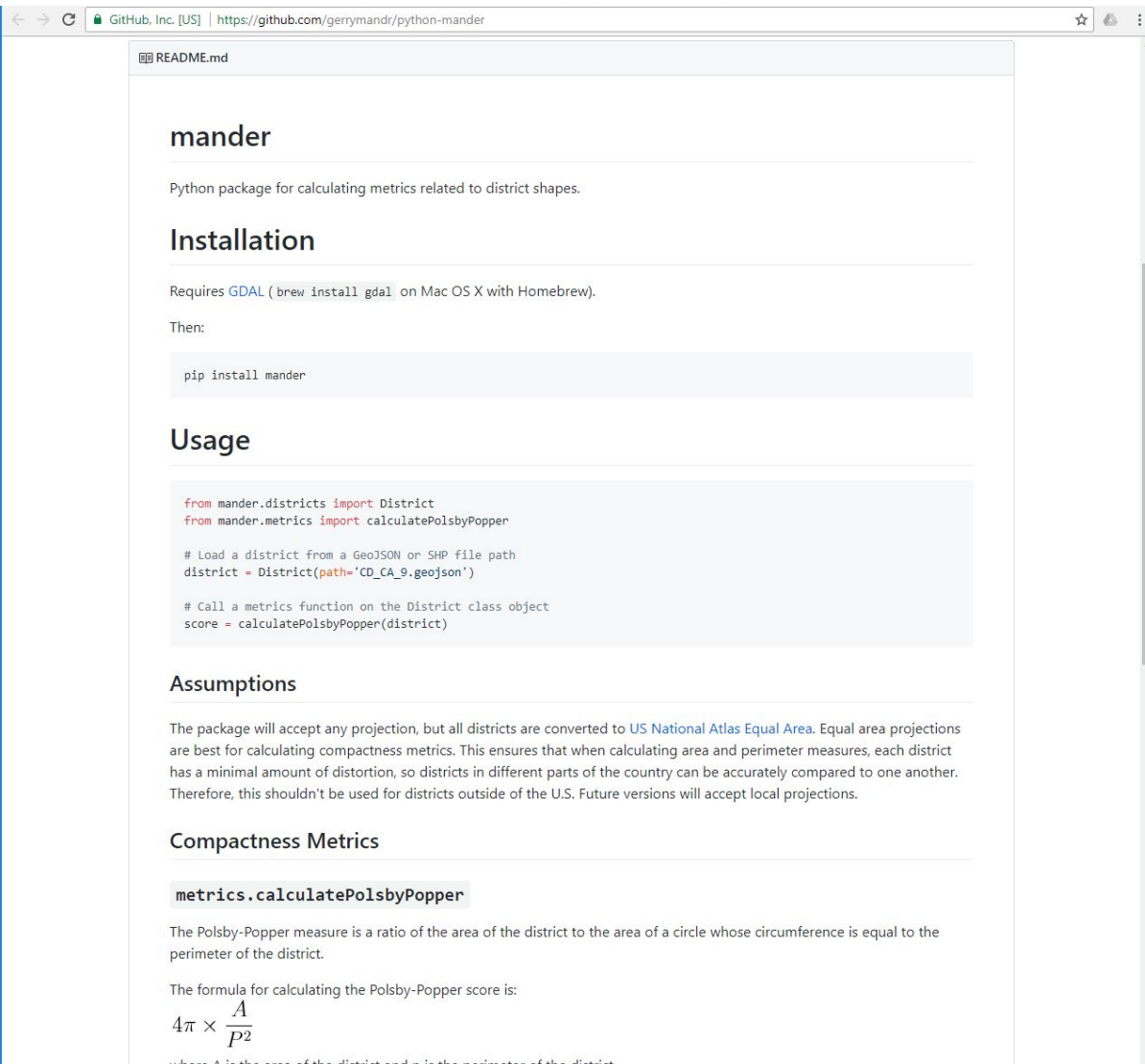
1 feature(s) selected on layer cb_2016_us_cd115_20m.

Coordinate: -11532824.3265348 Scale: 1:10,429,684 Magnifier: 100% Rotation: 0.0 Render EPSG:3785 (OTF)

<https://github.com/gerrymandr/qgis-compactness>

M. Gardner, R. Barnes,
A. Dennis, D. McGlone,
J. Connors

Mander & Compactnesslib



The screenshot shows a web browser displaying the README for the 'python-mander' package on GitHub. The page is titled 'MANDER' and describes it as a Python package for calculating metrics related to district shapes. It includes sections for 'Installation', 'Usage', 'Assumptions', and 'Compactness Metrics'. The 'Usage' section contains a code block showing how to load a district from a GeoJSON or SHP file and calculate the Polsby-Popper score. The 'Assumptions' section notes that the package uses the US National Atlas Equal Area projection. The 'Compactness Metrics' section explains the Polsby-Popper measure and provides the formula for calculating the score.

```
MANDER

Python package for calculating metrics related to district shapes.

Installation

Requires GDAL ( brew install gdal on Mac OS X with Homebrew).

Then:

pip install mander

Usage

from mander.districts import District
from mander.metrics import calculatePolsbyPopper

# Load a district from a GeoJSON or SHP file path
district = District(path='CD_CA_9.geojson')

# Call a metrics function on the District class object
score = calculatePolsbyPopper(district)

Assumptions

The package will accept any projection, but all districts are converted to US National Atlas Equal Area. Equal area projections are best for calculating compactness metrics. This ensures that when calculating area and perimeter measures, each district has a minimal amount of distortion, so districts in different parts of the country can be accurately compared to one another. Therefore, this shouldn't be used for districts outside of the U.S. Future versions will accept local projections.

Compactness Metrics

metrics.calculatePolsbyPopper

The Polsby-Popper measure is a ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district.

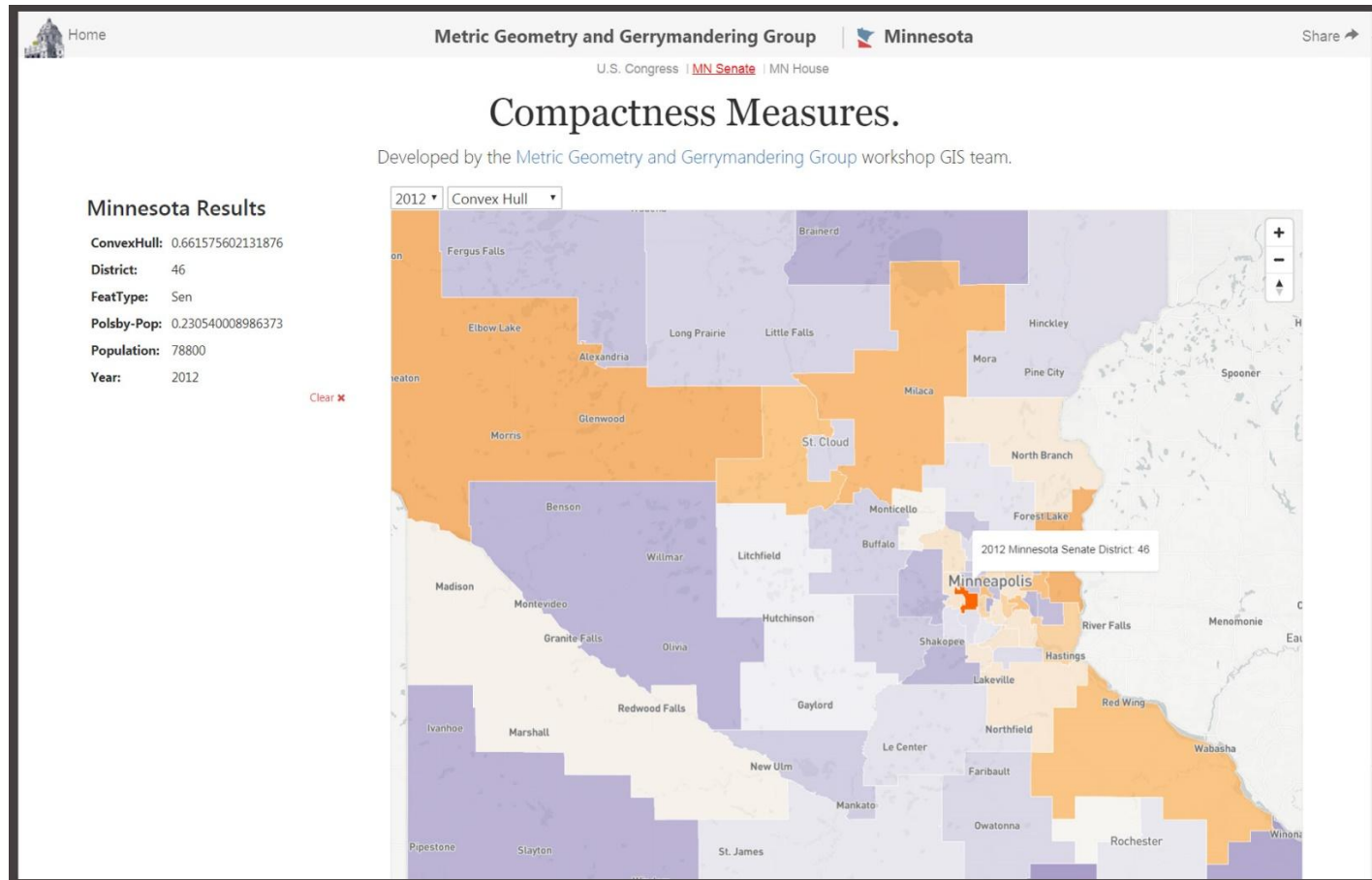
The formula for calculating the Polsby-Popper score is:

4π × A / P²

where A is the area of the district and P is the perimeter of the district.
```

<https://github.com/gerrymandr/python-mander>

Metric Visualizer



Future Workshops

- **Wisconsin**

October 12-15, 2017

- **North Carolina**

November 2-5, 2017

- **Texas**

February 1-4, 2018

- **California**

March 15-18, 2018

Potential Projects

- Unglamorous but necessary **data scraping**
- **Gerrymandr**, the app
- Demo and comparison of district **sampling** algorithms
- Illustrate **evolution** of districting plans
- Redistricting **competitions**
- GIS Team **request page**
- **Crowd-sourced** redistricting
- Many more!

Open Questions

- What is the role of **machine learning** in redistricting?
- How complicated is the **energy landscape** of political redistricting specifically?
- How do we ensure **transparency** for redistricting software?



Metric Geometry and Gerrymandering

Questions?